

## **General Disclaimer**

### **One or more of the Following Statements may affect this Document**

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

**NASA TECHNICAL  
MEMORANDUM**

**NASA TM X-73624**

**NASA TM X-73624**

(NASA-TM-X-73624) MINIMUM-TIME ACCELERATION  
OF AIRCRAFT TURBOFAN ENGINES (NASA) 16 p HC  
A02/MP A01 CSCL 21E

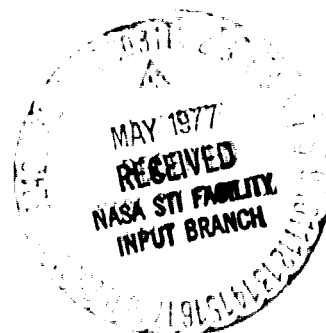
**N77-23112**

**G3/07 Unclass  
29072**

**MINIMUM-TIME ACCELERATION OF AIRCRAFT  
TURBOFAN ENGINES**

by Fred Teren  
Lewis Research Center  
Cleveland, Ohio 44135

TECHNICAL PAPER to be presented at the  
Joint Automatic Control Conference  
San Francisco, California, June 22-24, 1977



## MINIMUM-TIME ACCELERATION OF AIRCRAFT TURBOFAN ENGINES

FRED TEREN

National Aeronautics and Space Administration  
Lewis Research Center  
Cleveland, Ohio

### Abstract

Minimum-time accelerations of the F100 turbofan engine are presented. A piecewise-linear engine model, having three state variables and four control variables, is used to obtain the minimum-time solutions. The linear model which applies at a given time in the trajectory is determined by calculating a normalized "distance" from the current state to the equilibrium state associated with each linear model. The linear model associated with the closest equilibrium point is then used. The control histories for the minimum-time solutions are used as input to a nonlinear simulation of the F100 engine to verify the accuracy of the piecewise-linear solutions.

### INTRODUCTION

Modern, high-performance turbojet and turbofan engines are generally equipped with one or more variable geometry features in order to provide maximum propulsive efficiency over a range of engine power settings and flight conditions. For example, the J-85 engine (a one-spool turbojet used in the F5 aircraft) has variable inlet guide vanes, and variable bleeds in three stages of its eight-stage compressor. The TF30 engine (a two-spool turbofan, used in the F111 and F14 aircraft) has variable bleeds in the low and high compressors. The F100 engine (a two-spool turbofan, used in the F15 and F16 aircraft) has variable fan inlet guide vanes and variable compressor stator vanes. Each of these engines also has a variable-area exhaust nozzle and an afterburner. Variable area turbines, although not yet in operational use, have been tested on technology demonstrator engines.

Propulsive efficiency is probably the most important measure of an aircraft engine's performance. However, another important measure is the time required to accelerate from one thrust level to another. Engine acceleration is one of the functions of the engine control system, and may be accomplished via open-loop scheduling or closed-loop control. For each of the three engines referred to above, engine acceleration is accomplished by controlling fuel flow. The variable geometry features are not utilized in this function. Instead, these variables are scheduled in a steady-state manner to maximize propulsive efficiency.

In recent years, linear-quadratic regulator

theory has been developed for the design of multi-input multi-output control systems. An account of the theory and application is given, for example, in reference 1. Use of the theory has been facilitated by computer programs such as those described in references 2 and 3, which rapidly and efficiently calculate the optimal feedback control gains, given the system description and performance index. This theory has been applied recently to the design of control systems for aircraft gas turbine engines. In addition to the design of regulators, the problem of minimizing acceleration time has also been considered.

Michael and Farrar (refs. 4 and 5) applied linear quadratic regulator theory to the design of controls for the F401 turbofan engine. The nonlinear system equations were linearized about five different equilibrium points, a quadratic performance index intended to minimize acceleration time was formulated, and feedback control gains were determined. A nonlinear feedback control law was developed by curvefitting the resulting control gains as a function of compressor speed.

Weinberg (ref. 6) applied linear-quadratic regulator theory to the design of controls for the F100 engine. He showed that this engine could be adequately represented by three state variables - fan speed, compressor speed, and augmentor pressure. Four control variables were utilized, and linearized engine models were obtained at two equilibrium points. The problem of minimizing acceleration time was considered, and control system gains were derived by conducting small perturbation optimizations at each of two equilibrium points, using a quadratic performance index. The control gains were switched at a fixed value of fan speed, rather than varied in a continuous manner as in references 4 and 5.

In reference 7, Sevich and Beattie considered the minimization of acceleration time for a turbojet engine, using fuel flow and exhaust nozzle area as control variables. They used a quadratic performance index to approximate minimum-time solutions, as in references 4 to 6. However, they used a nonlinear engine model, rather than a series of linear models. The result was an open-loop, optimal trajectory. The controls were assumed to be piecewise constant, and the performance index was minimized by using a conjugate-gradient search technique.

Dehoff et al. (ref. 8) used linear-quadratic

regulator theory to design controls for the F100 engine. The control gains were generated using linear models with five state variables and four control variables at several equilibrium points. Principal emphasis was on the regulator design. Although acceleration control was considered, there was no specific attempt at minimizing the acceleration time.

References 4 through 8 all made use of integral, quadratic performance indices, in which both state and control deviations from some desired trajectory were penalized. The coefficients of the penalty terms were adjusted in an attempt to minimize the acceleration time. However, none of these reports can claim their final histories produce truly minimum-time accelerations.

In this paper, minimum-time acceleration histories are presented for the F100 turbofan engine. Four control variables, i.e., fuel flow, exhaust nozzle area, inlet guide vane position, and compressor stator vane position, are utilized. A piecewise-linear engine model having three state variables and four control variables was used to obtain the minimum-time solutions (ref. 9). The linear models were obtained by linearization of a nonlinear model at four sea-level, static equilibrium points, and were taken from reference 10. The linear model which applied at a given time in the trajectory was determined by calculating a normalized "distance" from the current state to the state at each of the equilibrium points; the linear model associated with the closest equilibrium point was then used. Linear state/control constraints which correspond to speed, temperature, pressure, and control limits were considered. The minimum-time solutions are used as inputs to a nonlinear computer simulation of the F100 engine (ref. 11) to verify the accuracy of the piecewise-linear solutions.

The minimum-time solutions were obtained by using a new algorithm, which is described in references 9 and 12. A more complete discussion and presentation of the results, as well as the relevant mathematical formulation, may be found in reference 9.

#### ENGINE MODEL

In a recent contractual effort under joint Air Force/NASA sponsorship (ref. 8), Systems Control Inc. (SCI) used linear quadratic regulator theory to design controls for the F100 engine. Linear models having 16 states and reduced-order models having five states were provided to SCI by Pratt & Whitney Aircraft (P&WA) for a number of equilibrium points at different flight conditions and power settings (PLA). Some of these models are given in reference 10. Four reduced-order, sea-level static models from the P&WA/SCI study, (PLA = 36, 52, 67, and 83 degrees) were used to generate the results presented herein.

In order to determine if the fifth-order models could be further reduced, the eigenvalues and eigenvectors of the fifth-order models were calculated. It was found that there were three

dominant eigenvalues, one real and one complex pair. The other two eigenvalues had much larger values (higher frequency). Therefore, the number of states was reduced from five to three, as described in reference 9.

#### TRANSIENT PERFORMANCE

The problem of minimizing the time required to accelerate the F100 engine from one equilibrium thrust level to another is considered. In solving this problem, a piecewise-linear model of the F100 engine, operating at the sea-level, static condition, was constructed from the four equilibrium linear models at PLA = 36, 52, 67, and 83 degrees. At any given time, the model whose equilibrium state is "closest" to the actual state at that time is used to represent the engine.

During an acceleration from near-idle to intermediate thrust, the engine is first represented by the PLA = 36° model. As the engine accelerates, the model switches successively to the 52, 67, and 83 degree models.

There are path inequality constraints which must be satisfied along a trajectory. Some of these constraints correspond to engine physical limits, others to control mechanical limits. The following constraints are assumed for this paper.

- (1) Turbine inlet temperature cannot exceed the equilibrium value at intermediate thrust by more than 50 degrees R.
- (2) Fan and compressor speeds cannot exceed the equilibrium values at intermediate thrust by more than 50 rpm.
- (3) Fan and compressor surge margins must not be less than 5 percent.
- (4) Inlet guide vanes, compressor vanes, exhaust nozzle area and fuel flow rate must not exceed their limits.

#### STATEMENT OF THE PROBLEM

The problem to be solved is to find the values of the four control variables, as functions of time, which minimize the "terminal error" (distance from the final state to the desired final state) for a specified acceleration time. This must be accomplished while satisfying the system equations and path constraints. A sequence of solutions to such problems may be used to find the minimum-time solution for a given value of terminal error. The mathematical problem formulation, necessary conditions for optimality and solution procedure may be found in reference 9.

#### RESULTS

The problem of minimizing the terminal error for an acceleration from near-idle (PLA = 24°) to intermediate thrust in 0.75 second was considered in reference 9. The responses of the problem variables (states, outputs, and controls) for the optimal solution are shown in figure 1. The state

variables (fan speed, compressor speed, and augmentor pressure) are shown in figures 1(a) to (c), respectively. It can be seen that the states approach the desired final values smoothly and with no overshoot.

The output variables are shown in figures 1(d) to (j). Because a piecewise-linear model has been utilized, the outputs are in general discontinuous at model switching points as well as at points of discontinuous control. If the piecewise linear model is a good representation of the engine, the discontinuities in the outputs at model switching points should be small. In figure 1, the discontinuities in thrust, low-pressure turbine inlet temperature, and airflow are so small that they are not visible. However, the discontinuities are substantial for fan and compressor surge margins and for combustor pressure.

The optimal control strategy results in the high-pressure turbine inlet temperature having its maximum value for the entire trajectory; this is shown in figure 1(d). Fan and compressor surge margins (figs. 1(e) and (f)) remain well above acceptable minimums. Low-pressure turbine inlet temperature (fig. 1(g)) is very nearly constant. Thrust (fig. 1(h)) increases smoothly and monotonically.

The optimal control histories are shown in figures 1(k) to (n). Fuel flow jumps initially from its idle value, then increases slowly. The optimal value of nozzle area is constant, and equal to its lower mechanical limit. For the acceleration shown in figure 1, the values of inlet guide vane position (IGV) and compressor variable vane position (HVS) were constrained to be within  $\pm 7$  degrees of the bill-of-material (BOM) control scheduled values. This was necessary because of limited off-schedule test data and to avoid violating flutter boundaries. For this case, IGV switched from 7 degrees below the scheduled value to 7 degrees above the scheduled value at 0.56 second; the value of HVS was 7 degrees below the schedule for the entire trajectory.

Figure 2 shows the terminal error as a function of acceleration time for accelerations from near-idle ( $PLA = 24^\circ$ ) to intermediate thrust ( $PLA = 83^\circ$ ). A terminal error of 0.10, for example, indicates that all three states are within 10 percent of their desired final values at the completion of the trajectory. Results are presented for values of IGV and HVS which are fully optimized, scheduled, and scheduled  $\pm 7^\circ$ . Figure 2 shows, for example, that an acceleration time of 0.80 second is required to reduce the terminal error to 0.05 when scheduled IGV and HVS are used. However, when the IGV and HVS are controlled optimally within  $\pm 7$  degrees of the scheduled values, the acceleration time is reduced to 0.74 second. Fully optimized IGV and HVS resulted in an acceleration time of 0.65 second. The corresponding trajectories have the same characteristics as shown in figure 1. In particular, the high-pressure turbine inlet temperature has its maximum value throughout each of the trajectories.

#### COMPARISON OF NONLINEAR AND PIECEWISE-LINEAR RESULTS

In order to determine the accuracy of the piecewise-linear solutions, the control histories for the time optimal solutions were used as input to a nonlinear F100 engine simulation (ref. 11). Figure 3 presents a comparison of nonlinear and piecewise-linear transient responses. The control variable histories are based on an optimal acceleration from  $PLA = 24$  degrees to intermediate thrust, with a specified acceleration time of 0.8 second. For this case, the values of IGV and HVS were constrained to be within  $\pm 7$  degrees of the scheduled values. It can be seen that the nonlinear and piecewise-linear responses of compressor speed and augmentor pressure are in good agreement. However, differences are observed in the fan speed responses. Also, there are substantial and important differences in the high-pressure turbine inlet temperature and fan and compressor surge margin responses. The nonlinear results show that the maximum value of the high-pressure turbine inlet temperature is violated by a large amount, and that the compressor surges at about 0.06 second. The fan does not surge, but the fan surge margin does fall below the minimum value of 5 percent early in the trajectory.

There are several possible explanations for the differences between nonlinear and piecewise-linear results observed in figure 3. They are as follows:

- (1) The individual linear models may not be precisely correct even for small perturbation inputs.
- (2) The number of equilibrium linear models may be inadequate to accurately describe the system nonlinearities with respect to the state variables.
- (3) There may be substantial nonlinearities with respect to the control variables, which are not included in the piecewise-linear model.

- (4) Model reduction from sixteenth order to third order may have resulted in modeling inaccuracies.

An investigation is currently underway to determine and eliminate the cause of the piecewise-linear model inaccuracies.

#### COMPARISON OF MINIMUM-TIME AND BOM CONTROL RESPONSES

It is of interest to compare the transient responses obtained by using optimal, minimum-time strategy with those obtained by using the BOM control. Such a comparison is made in figure 4, for an acceleration from  $PLA = 24$  degrees to intermediate thrust. The minimum time controls were obtained by using the piecewise-linear model with the values of IGV and HVS constrained to be within  $\pm 7$  degrees of the BOM scheduled values. However, the results presented in figure 4 were obtained by using the minimum-time and BOM controls in con-



junction with the nonlinear F100 engine simulation. The results show that the minimum-time control strategy produced a more rapid acceleration to intermediate thrust. It appears that the principal reason for the improved acceleration is the much more rapid increase in fuel flow, which results in a rapid increase in high-pressure turbine inlet temperature. Naturally, the comparison of performance is invalidated because of the violation of constraints which occurs. Nevertheless, it seems highly probable that substantial improvement in acceleration time can be made without violation of engine constraints since the high-pressure turbine inlet temperature (fig. 4(d)) increases very slowly when the BOM control is used.

#### CONCLUDING REMARKS

In this paper, minimum-time acceleration histories are presented for the F100 engine. A piecewise-linear engine model, having three state variables and four control variables, was used to obtain the minimum time solutions. The resulting control histories are used as inputs to a nonlinear simulation of the F100 engine to verify the accuracy of the piecewise-linear solutions.

A comparison of the nonlinear and piecewise-linear solutions revealed significant differences in some of the engine responses. Several possible explanations for these differences were noted; this problem is currently under investigation.

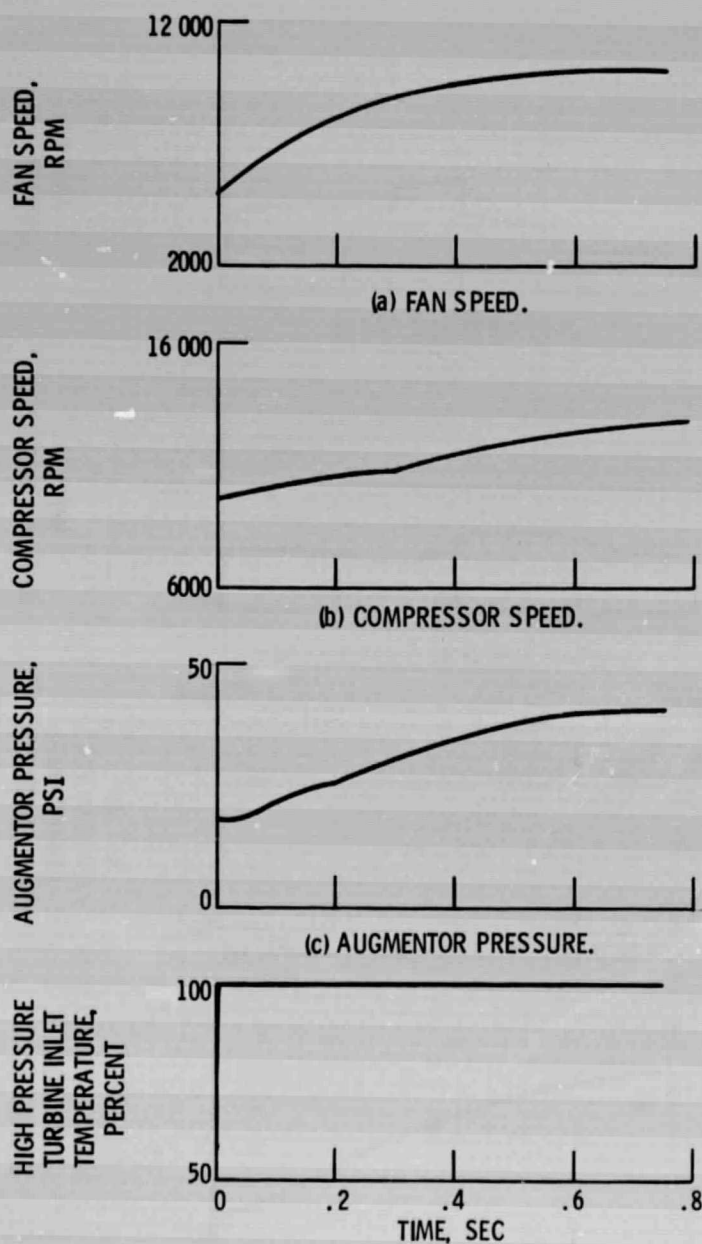
In this paper, it was assumed that the control variables could jump discontinuously from one value to another. Studies are currently underway to find minimum-time acceleration histories which satisfy the control variable rate limits in addition to the other constraints.

Results presented herein indicate that improved steady-state and transient performance may be obtained by using optimal control strategy. Such strategy sometimes calls for operation of the controls in a manner which has not been previously tested experimentally, even though the nonlinear simulation contains equations for predicting the effect of such control action. Further experimental testing is indicated in order to systematically explore the steady-state and transient effects of off-scheduled control action, and to determine if the predicted performance gains can be achieved.

#### REFERENCES

1. A. E. Bryson, Jr., "Control Theory for Random Systems", Stanford University, Rept. SUDAAR-447, 1972; also NASA CR-132054.
2. A. E. Bryson and W. E. Hall, "Optimal Control and Filter Synthesis by Eigenvector Decomposition", Stanford University, Rept. SUDAAR-436, 1971.
3. L. C. Geyser and B. Lehtinen, "Digital Program for Solving the Linear Stochastic Optimal Control and Estimation Problem", NASA TN D-7820, Mar. 1975.
4. G. J. Michael and F. A. Farrar, "Development of Optimal Control Modes for Advanced Technology Propulsion Systems", United Aircraft Corp., Rept. UARL-N911620-2, 1974.
5. G. J. Michael and F. A. Farrar, "An Analytical Method for the Synthesis of Nonlinear Multi-variable Feedback Control", United Aircraft Corp., Rept. UARL-M941338-2, 1973.
6. M. S. Weinberg, "A Multi-Variable Control for the F100 Engine Operating at Sea Level Static", Aeronautical Systems Division, Rept. ASD-TR-75-28, 1975.
7. G. T. Sevich and E. C. Beattie, "Integrated Flight/Propulsion Control Design Techniques Starting with the Engine", SAE Air Transportation Meeting, Dallas, Texas, Apr. 30-May 2, 1974, Paper 740481.
8. R. L. DeHoff and W. E. Hall, "Multivariable Design Procedures for the F100 Turbofan Engine", Systems Control, Inc. (Vt), Palo Alto, Calif., Rept. F33615-75-C-2053, 1977.
9. F. Teren, "Minimum Time Acceleration of Aircraft Turbofan Engines by Using an Algorithm based on Nonlinear Programming", Depart. of Aeronautics and Astronautics, Stanford University, Stanford, California, to be published.
10. R. J. Miller and R. P. Hackney, "Research on F100 Multivariable Control", Pratt and Whitney Aircraft, Rept. FR-7809, 1976.
11. "F100-PW-100(3) Transient Engine Simulation Deck", Pratt and Whitney Aircraft, Rept. FR-6014, 1973.
12. F. Teren, "Solution of Transient Optimization Problems by Using an Algorithm Based on Nonlinear Programming," JACC Conference Paper, San Francisco, Calif., June 22-24, 1977.

E-9115



(d) HIGH-PRESSURE TURBINE INLET TEMPERATURE.

Figure 1. - Acceleration from PLA = 24 deg to intermediate thrust. Acceleration time, 0.75 sec. IGV and HVS scheduled  $\pm 7$  deg.

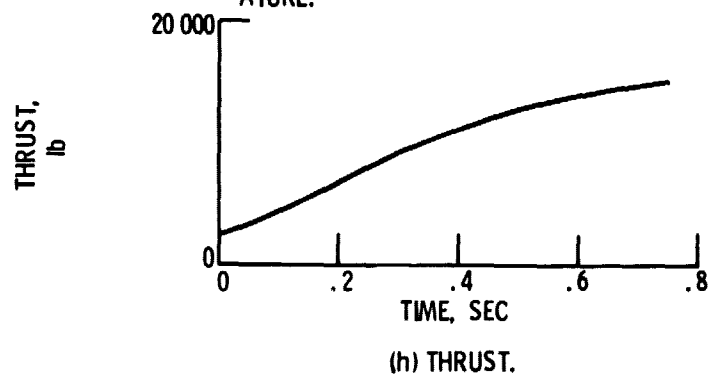
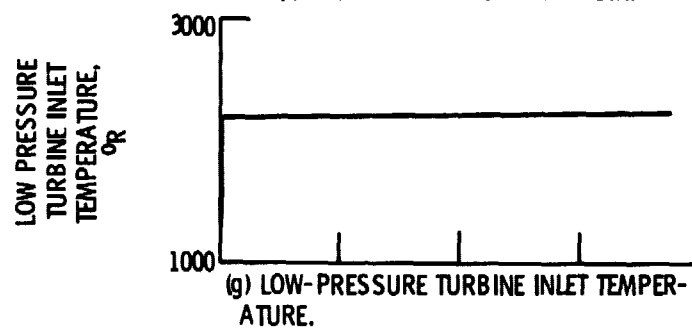
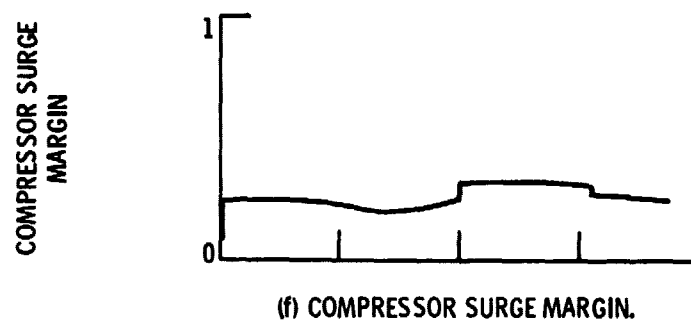
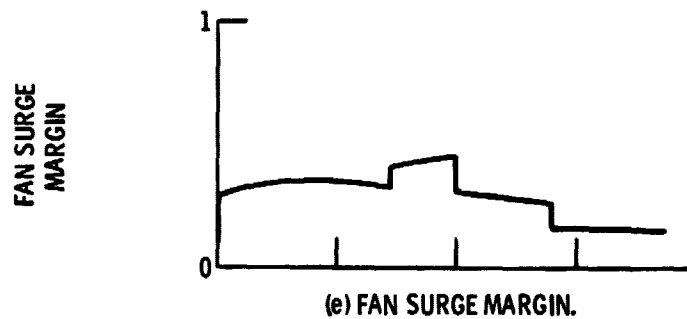


Figure 1. - Continued.



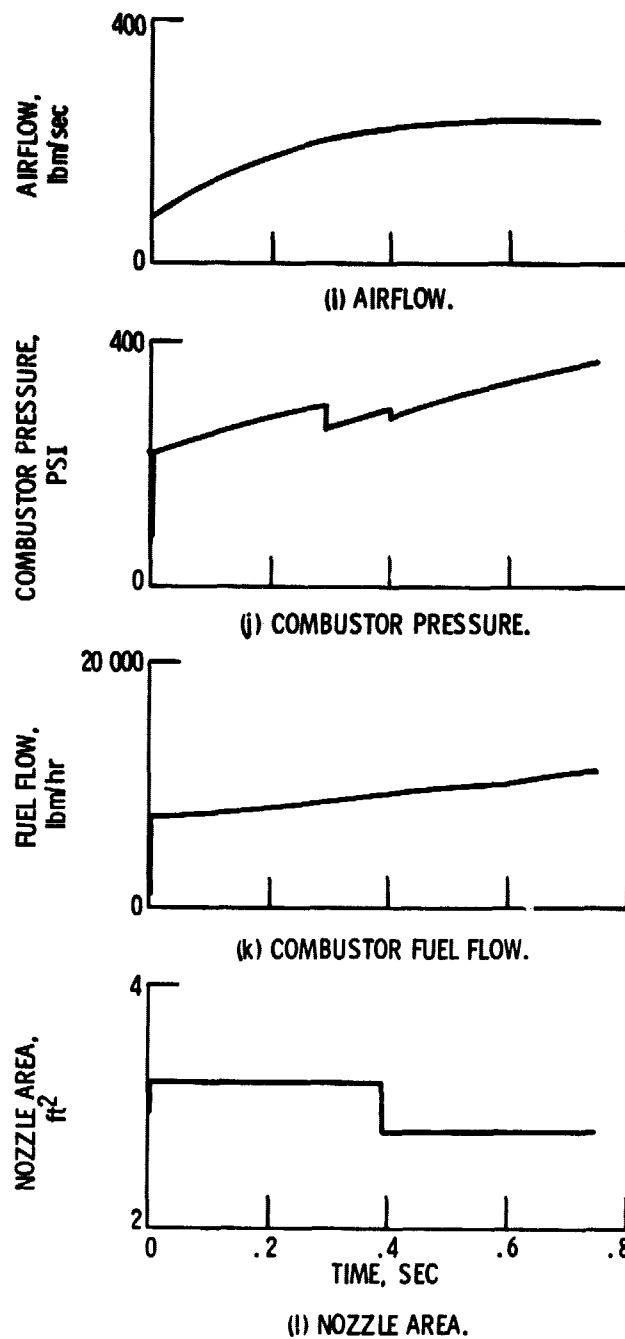


Figure 1. - Continued.

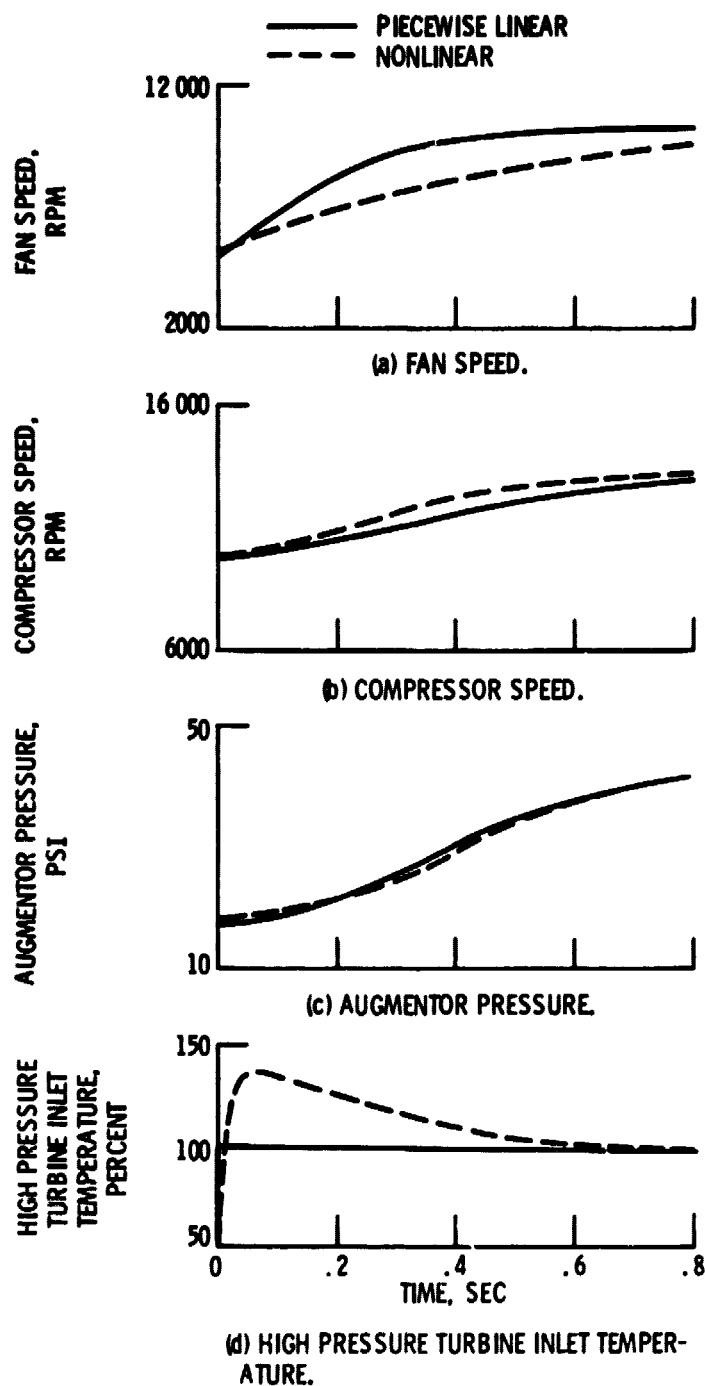
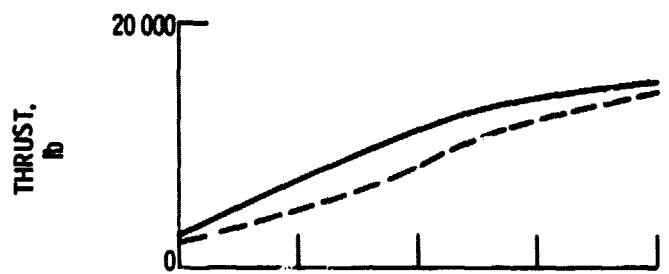
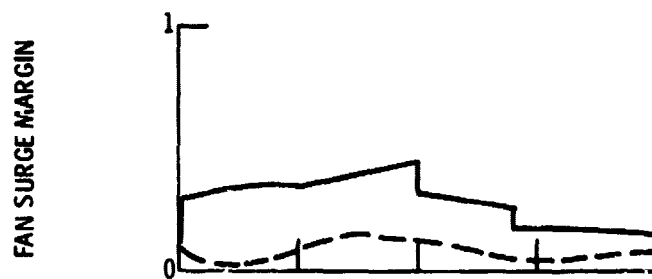


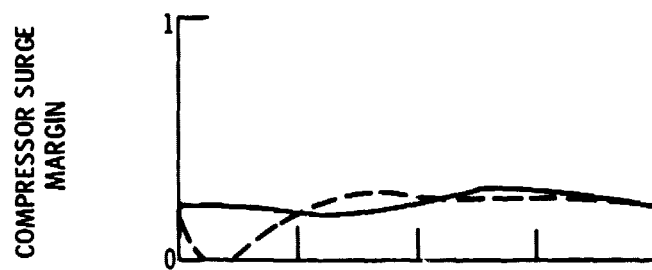
Figure 3. - Comparison of nonlinear and piecewise-linear results. Acceleration from PLA = 24 deg to intermediate thrust. Acceleration time, 0.8 sec. IGV and HVS, scheduled  $\pm 7$  deg.



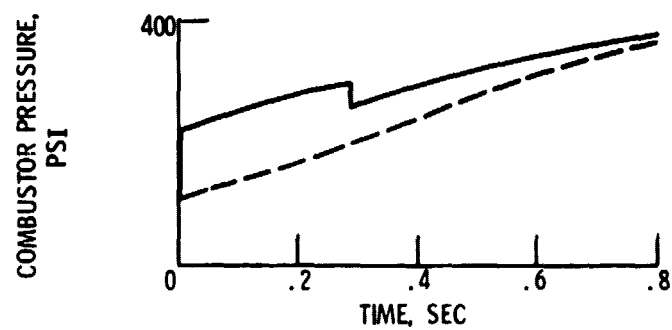
(e) THRUST.



(f) FAN SURGE MARGIN.



(g) COMPRESSOR SURGE MARGIN.



(h) COMBUSTOR PRESSURE.

Figure 3. - Continued.

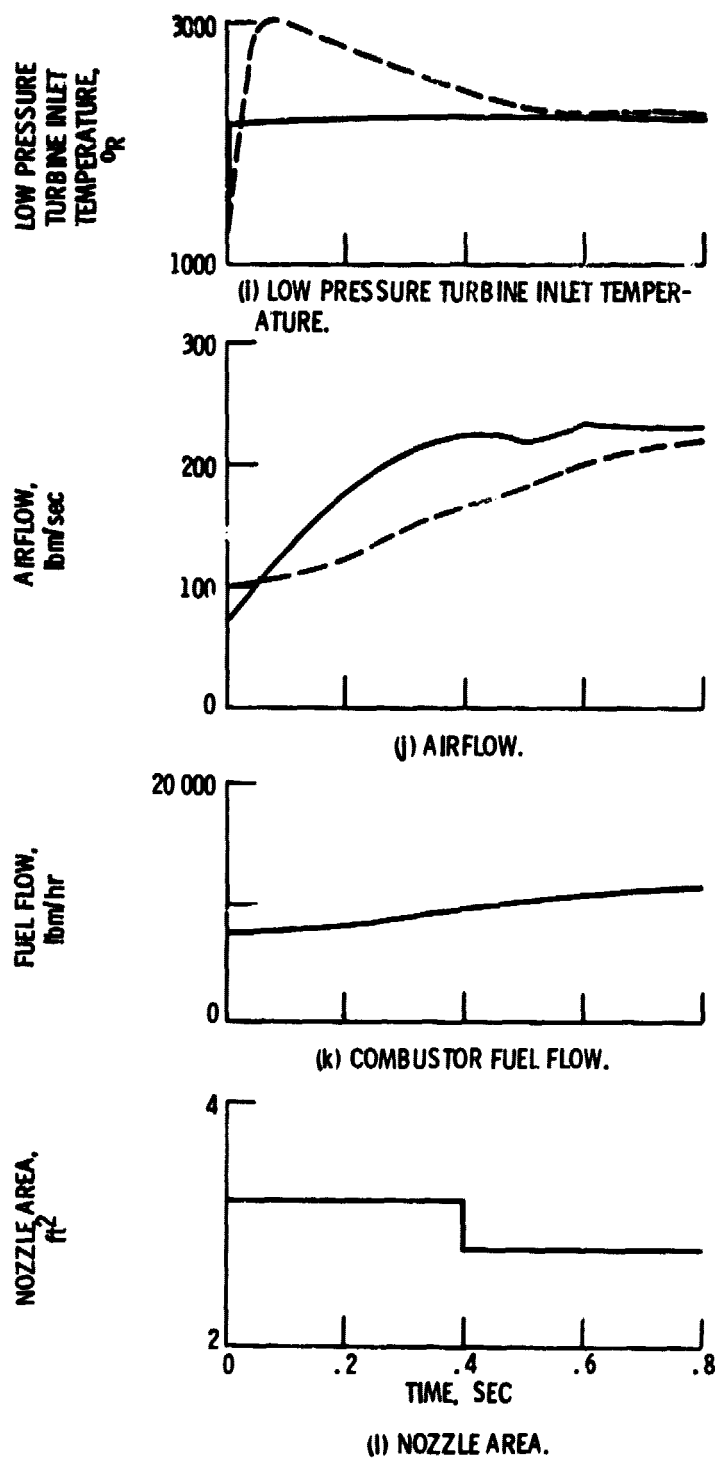
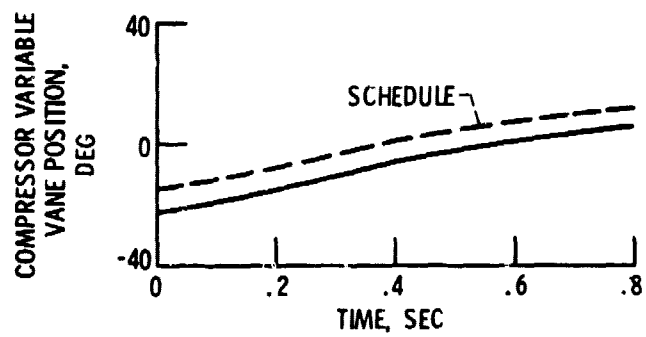
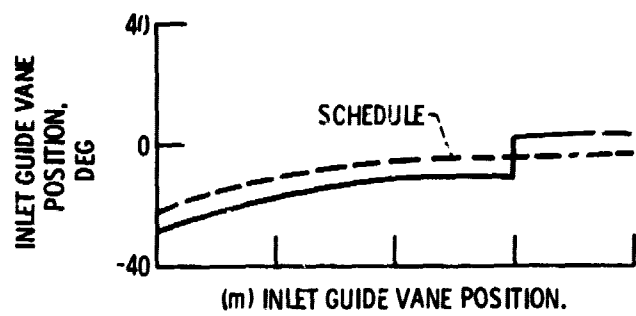


Figure 3. - Continued.



(n) COMPRESSOR VARIABLE VANE POSITION.

Figure 3. - Concluded.

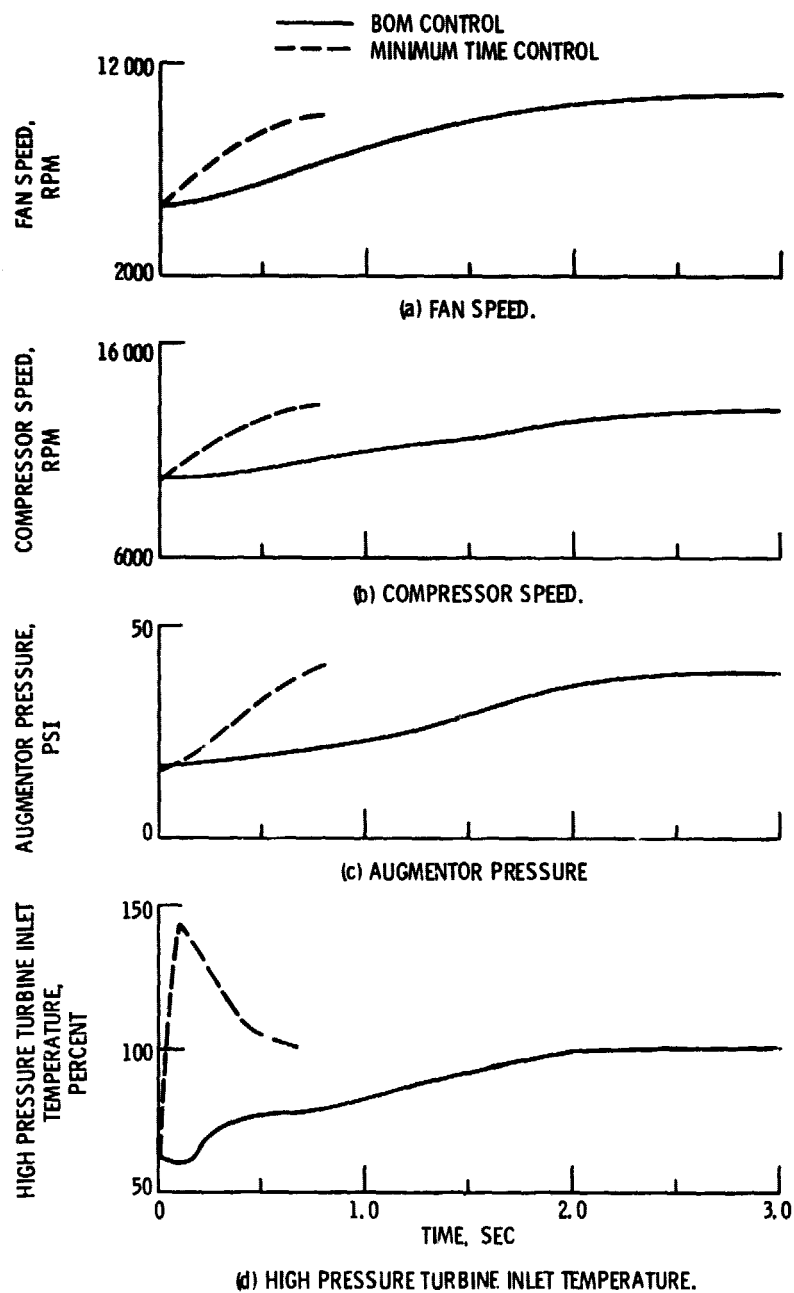
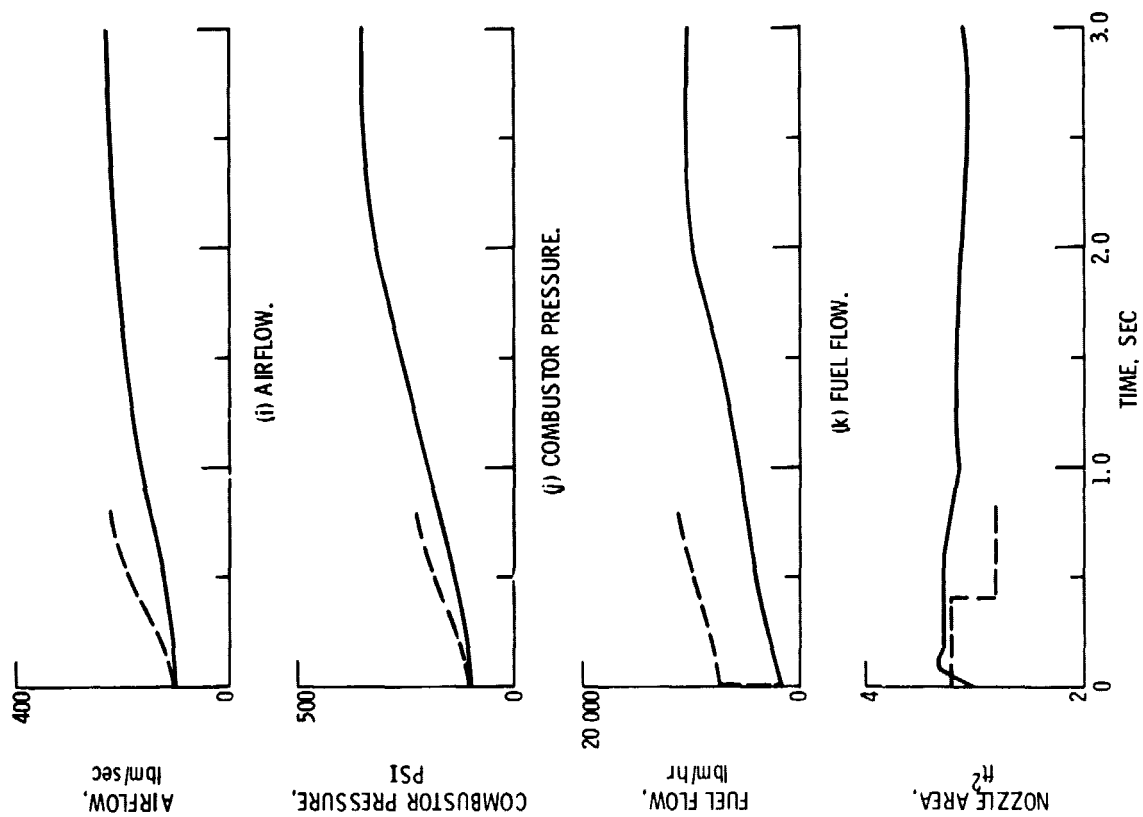


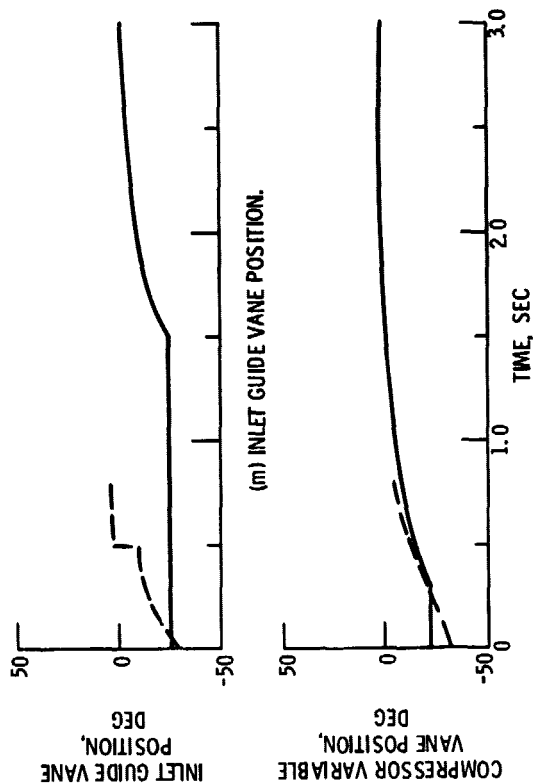
Figure 4. - Comparison of trajectories for minimum time and BOM control. Acceleration from PLA = 24 deg to intermediate thrust. Nonlinear simulation.





(l) NOZZLE AREA.

Figure 4. - Continued.



(n) COMPRESSOR VARIABLE VANE POSITION.

Figure 4. - Concluded.